5-person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper for each numbered problem**. Place your team letter in the upper right corner of each page that will be turned in (failure to do this will result in no score). Place problem numbers in the upper left corner. Problems are equally weighted; **teams must show complete solutions (not just answers) to receive credit**. More specific

5. The digits of 2012 sum to 5. Since the year one, in how many years have the digits of the year summed to five? (Remember to fully justify your answer!)

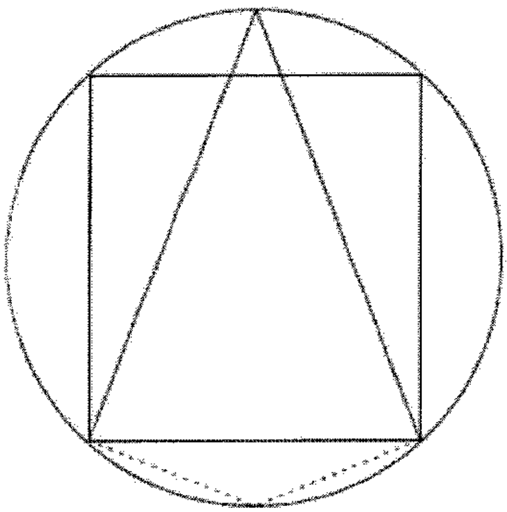
6. Given four vertices, a "graph" is created by joining any number of the vertices by straight line(s). Two [illegible]

5-person Team Test - Solutions

For DAY and MAY to be even either the AY matched or nothing matched. If the AY matched then the BUY

says that B_Y matches and the word is BAY. But then SAY is not Odd. So niether the A nor the Y match. So we get S must match from SAY. From the other two Odds we get either SEE or SUN. But BUY is even and so only SEE is left as a possibility.

3. Inscribe a rectangle of base b and height h in a circle of radius 1, and inscribe an isosceles triangle in a region of the circle cut off by one base of the rectangle (with that base becoming also one side of the triangle). For what value(s) of h can one construct these shapes as described such that the rectangle and triangle have the same area?



   This can be interpreted in two ways, either as the solid triangle shown in the picture or as the triangle formed from the two dotted lines and the base of the rectangle.

   *Let **b** be the base of the rectangle and **h** be its height.*

   Using the triangle including the dotted lines, since the diameter is 2, the height of the triangle is $\frac{1}{2}(2-\mathbf{h}) = 1-\mathbf{h}/2$. *Thus the area of the triangle is* $\frac{1}{2}(\mathbf{b})(1-\mathbf{h}/2)$. *Setting this equal to the area of the rectangle **bh**, we find* $\mathbf{h} = 2/5$.

   Using the triangle including the solid lines, the height will be $1+\mathbf{h}/2$ *since the radius is one. Thus the area of the solid triangle is* $\frac{1}{2}(\mathbf{b})(1+\mathbf{h}/2)$. *Setting this equal to **bh** yields* $\mathbf{h} = 2/3$.

   The possible values of H are 2/3 and 2/5.

4. A weight hangs on a rope of length 100 feet as shown in the diagram below. The rope hangs with an angle of 50 degrees from the horizontal on Beth's end and 35 degrees from the horizontal on Fred's end. How far below the level of Beth and Fred is the weight?

   *Let **h** be the distance we are looking for. Let **x** be the length of rope from Fred to*

[redacted]

five? (Remember to fully justify your answer!)

Assume a four-digit year. If the thousands digit is zero, then the other three digits sum to five. This means they are 500, 410, 320, 311, or 221. There are three ways to arrange 500, 311, and 221 and six ways to [redacted]